Study on the influence of design parameter variation on the dynamic behaviour of honeycomb sandwich panels

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Abstract: **Sandwich panels are layered structures that consist of at least five layers : two thin face sheets that are bonded with bonding layers to the thick core. The core has a very low density whereas the face sheets are stiff and strong. The entire panel combines high mechanical properties with a very low areal mass. Most of the structural characteristics of the panel (material selection and thickness of each layer) can be selected independently of other parameters, and the overall characteristics of the panel depend on the particular selection of parameters. Because of the wide range of panel parameters, numerical modelling is useful to provide insight into the structural characteristics of a particular panel. This paper studies the effect of design parameter variations on the dynamic behaviour of honeycomb sandwich panels. The dynamic behaviour includes natural frequencies, mode shapes and damping of such panels with free boundary conditions. In the first section the structure of honeycomb sandwich panels is illustrated, in particular those with a ThermHex core. For a typical honeycomb panel the different design parameters are outlined.**

Natural frequencies and mode shapes can be predicted approximately using analytical models. Some of the methods are outlined in this article.

The second section of the paper presents the numerical modelling of a sandwich panel using commercial finite element codes. Different core modelling strategies are compared, e.g. geometrically correct or as a homogenised equivalent material. Advantages and drawbacks of the different methods are outlined. Different ways of modelling damping in the panels are also presented.

The third section discusses the experimental validation. To validate the finite element models, measurements are carried out on some test panels. Free-free boundary conditions are provided by elastically suspending the panels. To make measurements totally contactless, the test panels are excited acoustically and the vibration measurement is performed with a laser vibrometer. The way the data are captured and processed is also outlined.

Measured natural frequencies and mode shapes are compared with the calculated results from the different FE models and the analytical models. The techniques that are used for this comparison are briefly discussed.

The different FE models are updated using results from a sensitivity analysis. This analysis is performed theoretically for every design parameter and is discussed in detail. Results from the updated models are again compared with those obtained from measurements.

The uncertainty on different design parameters is studied and discussed. The influence of these various uncertainties on the natural frequencies and mode shapes is investigated using Monte Carlo simulations.

Keywords: **honeycomb sandwich panel, design parameter uncertainty, dynamic behaviour**

I. INTRODUCTION

Honeycomb sandwich panels consist of a thick honeycomb core that is bonded to thin face sheets. The structure of such a panel is shown in fig. 1. The coordinate system is used throughout this text, although the axes can also be indicated with numbers 1 to 3. The panels discussed in this article are built up with a ThermHex core. This type of honeycomb core is fabricated according to the folded honeycomb concept.

During this process a thermoplastic sheet is successively cut, folded and glued to form a completely closed honeycomb core. This process is shown in fig. 2.

The test panels that are discussed here have a core that is bonded to thin sheets of galvanised steel.

The elastic mechanical properties of a typical honeycomb core are described and analytically calculated by Gibson & Ashby [2]. They propose formulas for calculation of the in-plane and out-of-plane elastic moduli and Poisson ratios of the core.

As honeycomb sandwich panels become more and more important as structural parts in the automotive and aerospace industry, the need for accurate modelling of the dynamic behaviour of such panels increases. Accurate modelling requires knowledge of the different design parameters that determine the dynamic behaviour, which in this case are natural frequencies and mode shapes.

The main work on the dynamics of sandwich panels is related to conventional foam-core structures. Little work has been carried out on honeycomb panels. Nilsson & Nilsson [3] tried to analytically predict natural frequencies of a honeycomb sandwich plate with free boundary conditions using Blevins [4] formula in which areal mass and equivalent bending stiffness are frequency dependent.

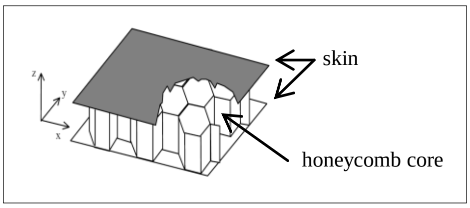

Fig. 1. Honeycomb sandwich panel

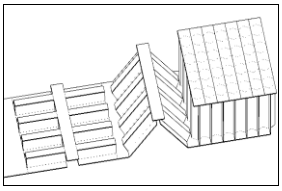

Fig. 2. The folded honeycomb process for completely closed honeycombs.

Another way to predict natural frequencies and mode shapes of a honeycomb panel is by means of the finite element method. In the past years, different new approaches have been developed which incorporate high order shear deformation of the core. Work in this area has been carried out by Topdar [5] and Qunli Liu [6][7]. The latter stated that the shear moduli of the core are important factors in the determination of the values of the natural frequencies and the sequence of mode shapes, especially at high frequencies. At low frequencies natural frequencies are mostly determined by the bending stiffness of the panel.

II. Considered design parameters

Honeycomb panels are complex structures with a high number of design parameters. It is therefore difficult to accurately predict their dynamic behaviour, certainly when some of the parameters are very difficult or even impossible to measure in a direct way. When the design parameters are studied one has to mention whether the honeycomb core of the model used is homogenized or not.

For the general honeycomb panel structure shown in fig. 1 table 1 gives an overview of the different design parameters studied in this article. They can be divided into two groups, geometric and material parameters. The abbreviations for the different parameters will be used throughout the article.

TABLE I

Studied design parameters for a panel with a non homogenized Thermhex core and isotropic skins.

	parameter description	symbol	unit
geometry	overall panel width	w	mm
	overall panel length	l	mm
	skin thickness	t_s	mm
	core thickness	t_c	mm
	cell wall thickness	t	mm
	cell size	D	mm
material	core material elastic modulus	E_c	MPa
	core material poisson ratio	μ_c	/
	core material density	ρ_c	kg/m³
	skin elastic modulus	E_c	MPa
	skin poisson ratio	μ_s	/

In table 1 the cell size D is the diameter of the circumscribing circle, minus the cell wall thickness, of a regular hexagonal cell. Note that, at this stage, no parameters concerning the bonding layer between core and skin are considered. In a first approach the glue is considered as a perfect rigid connection of core and skin.

It is obvious from fig. 1 and 2 that a honeycomb core has 3 planes of symmetry, hence it can be considered as an orthotropic material. In that case the elastic behaviour of the homogenised core is determined by 9 independent elastic constants. Note that the Thermhex core shown in fig. 2 is in fact a three layer material. The bonding layer between core and skins is now a uniform layer. In fact, a honeycomb sandwich panel that is built up with two isotropic skins and a Thermhex core can be seen as a 7 layer laminate. Table 2 gives an overview of the design parameters of such a laminate, studied in this article. As in table 1 the parameters can here be divided in the same two groups.

TABLE 2

Studied design parameters for a panel with a homogenized Thermhex core and isotropic skins.

	parameter description	symbol	unit
geometry	overall panel width	w	mm
	overall panel length	l	mm
	skin thickness	t_s	mm
	core thickness	t_c	mm
	outer core layer thickness	t_{cl}	mm
	bonding layer thickness	t_b	mm
material	core elastic modulus 1	E_{c1}	MPa
	core poisson ratio 12	μ_{c12}	/
	core shear modulus 12	G_{c12}	MPa
	core elastic modulus 3	E_{c3}	MPa
	core poisson ratio 13	μ_{c13}	/
	core shear modulus 13	G_{c13}	MPa
	core elastic modulus 2	E_{c2}	MPa
	core poisson ratio 23	μ_{c23}	/
	core shear modulus 23	G_{c23}	MPa
	equivalent core density	ρ_{ec}	kg/m³
	glue elastic modulus	E_b	MPa
	glue poisson ratio	μ_b	
	glue density	ρ_b	kg/m³
	skin elastic modulus	E_s	MPa
	skin poisson ratio	μ_s	/
	skin density	ρ_s	kg/m³
	outer core layer elastic modulus	E_{cl}	MPa
	outer core layer density	ρ_{cl}	kg/m³
	outer core layer poisson ratio	μ_{cl}	/

III. Finite element models

A. *Description of FE - methods used*

A geometrically accurate and realistic FE model of a sandwich panel with a honeycomb core inevitably has large numbers of nodes and elements. This article presents two different approaches to build a simplified model. The finite element software used for all models is Siemens UGS NX6.

A first way of modelling a honeycomb sandwich panel (with ThermHex core) is to fully model the panel's core geometry. This is illustrated in figure 3. Each cell wall is meshed with 8 (2x4) rectangular 4 node shell elements. The skin faces are meshed with triangular 3 node shell elements. For visibility only one meshed skin face is shown. For a panel with length 300 mm and width 200 mm the finite element model has over 117000 elements. It is obvious that the major drawback of this FE – approach is the high computational effort due to the high number of elements.

Another problem is that the uniform bonding layer between core and skin, and the uniform layer of core material are not modelled. The contribution of mass and stiffness to structural behaviour is not taken into account. The properties of these intermediate layers are not well known.

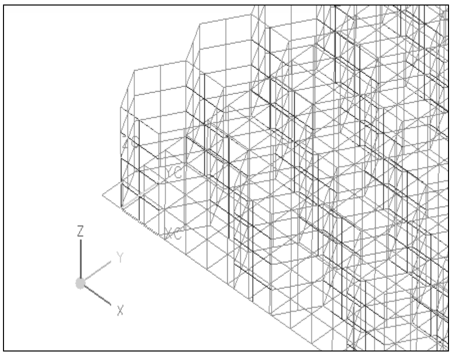

Fig. 3. Close-up of honeycomb panel model with fully modelled core.

A second way to model a honeycomb sandwich panel is by means of homogenisation of the core. As mentioned in the introduction a honeycomb core can be modelled as an orthotropic material. The 9 independent elastic constants are calculated by simulating tensile and shear tests on a sample of honeycomb core with 10 x 10 cells. The principle is shown in fig. 4.

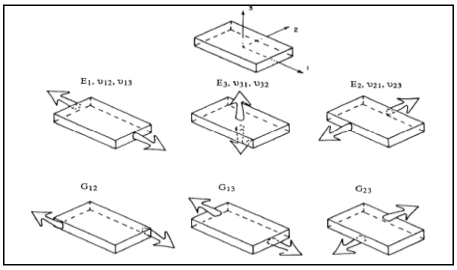

Fig. 4. Principle of determining the elastic constants of an orthotropic material.

The mass density of an equivalent homogenised regular honeycomb core is determined with equation (1) from Gibson & Ashby [2].

$$\frac{\rho^*}{\rho_s} = \frac{2 \cdot t}{\sqrt{3} \cdot l} \cdot \left(1 - \frac{t}{2 \cdot \sqrt{3} \cdot l}\right) \qquad (1)$$

In equation (1) ρ_s is the mass density of the honeycomb core material and ρ^* is the equivalent mass density of the homogenized core material. The cell wall thickness is expressed by t and l is the cell wall width. Due to the continuous folding process of the honeycomb core the cell walls parallel to the folding direction have a double thickness. For each hexagonal cell this counts for 2 walls.

Once these core parameters are determined the panel is modelled as a 7 layer laminate, thus including two uniform bonding layers. The elastic properties of the bonding layer are not known but they are estimated.

The whole panel is meshed with rectangular 8 node shell elements. For the same panel with dimensions 300 x 200 mm now only 121 elements are used.

From the FE models the first 10 natural frequencies and mode shapes are calculated for a panel with free-free boundary conditions. At this stage no damping is introduced in the models.

B. Comparison of the different FE - methods

The two FE models described in section IV.A are compared in this section. They are adopted for a honeycomb panel with dimensions and properties given in table 3. The face sheets are made from steel and the ThermHex core from polypropylene.

TABLE 3

DIMENSIONS AND PROPERTIES OF HONEYCOMB TEST PANEL.

parameter description	symbol	value
overall panel width	w	200 mm
overall panel height	l	300 mm
skin thickness	t_s	0,3 mm
core thickness	t_c	7,55 mm
cell wall thickness	t	0,18 mm
cell size	D	8 mm
bonding layer thickness	t_b	0,1 mm
core material elastic modulus	E_c	1500 MPa
core material poisson ratio	μ_c	0,39
core material density	ρ_c	1100 kg/m³
skin elastic modulus	E_c	210000 MPa
skin poisson ratio	μ_s	0,3
glue elastic modulus	E_b	10 MPa
glue density	ρ_g	1000 kg/m³
glue poisson ratio	μ_b	0,3

Figure 5 shows the corresponding modes and their frequencies, obtained by a geometrically realistic model of the core. both FE models. Figure 6 gives a comparison of the first 10 natural frequencies, calculated with both FE methods.

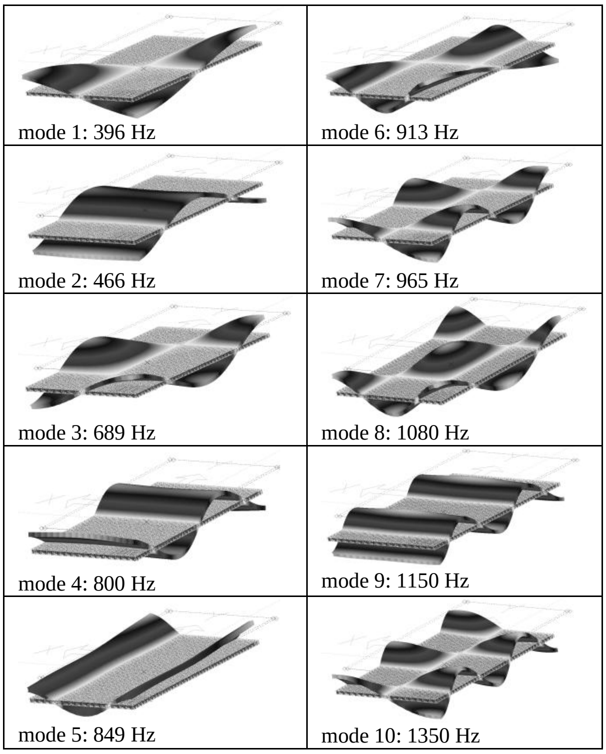

Fig. 5. First 10 calculated mode shapes, obtained by modelling the honeycomb core geometrically realistic.

The resemblance between the mode shapes of the two series, obtained by the different FE models, is checked visually. The mode sequence of both series is not identical. Only the natural frequencies of corresponding mode shapes are compared. This explains why in case of the 7 layered laminate, mode 5 has a lower natural frequency than mode 4. The relative deviations between both FE models varies between 2 and 17 %. The difference tends to increase as frequency increases.

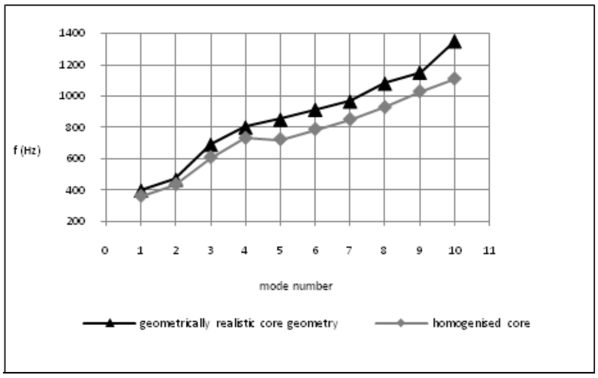

Fig. 6. Natural frequencies of the first 10 modes for FE models with geometrically realistic core geometry (black), and with homogenised core (grey).

The rather large differences between the two models have a number of reasons. In the model with the non-homogenised core the bonding layer is not modelled, thus making the panel stiffer than in the case where the bonding layer is considered. As the frequency increases, shear deformation of core an skins becomes more important. The fact that the bonding layer between core and skin is primarily shear loaded explains partly that the relative difference between the two methods increases with frequency. A second reason is that the cell walls parallel to the folding direction have a double thickness in the FE model. In reality these double cell walls are in fact 2 single walls, placed next to each other. This makes that the shear moduli of the modelled core are overestimated. As a result the modelled panel is stiffer as frequency increases.

C. *Modelling of damping*

With the proportional damping model, the damping matrix is calculated with the modal stiffness and modal mass matrices, using equation (2).

$$[C] = \alpha[K] + \beta[M] \qquad (2)$$

The mass and stiffness damping constants, α and β are determined by choosing the fractions of critical damping at two different frequencies and solving simultaneous equations for the constants. In this way, damping can be modelled as a linear function of frequency.

IV. Experimental Validation

A. *Experimental set-up*

In the experimental modal analysis a contactless measurement method is used. The panel is suspended from elastic wires to attain free-free boundary conditions. Its properties are given in table 3. A regular grid of 11 x 11 measurement points is marked on the panel. The panel is excited acoustically by a loudspeaker. The excitation signal used is random noise with a bandwidth of 1,6 kHz. The response of the panel is measured with a laser vibrometer. Measurements are performed in an anechoic room. A Bruel & Kjaer 7536 data system is used for data acquisition. Experimental frequency response functions are identified from the measured data.

B. *Processing measured data*

Figure 7 shows the summed FRF for the test panel described in table 3.

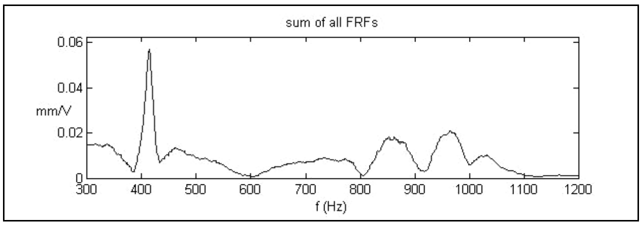

Fig. 7. Sum of all frequency response functions for the test panel described in table 3.

In figure 7 a number of peaks is visible in the chosen frequency band from 300 to 1200 Hz. Only the one at 414 Hz seems well isolated and thus the corresponding resonance frequency is easily readable. The other peaks are rather wide, making it difficult to visually detect a specific resonance frequency. This is the result of the coupling of two or more nearby modes, due to the high damping in the structure. The simple peak picking method should be used with caution.
Figure 8 shows the measured deflection shapes, corresponding to modes 2, 4, 5 and 8 from figure 5.

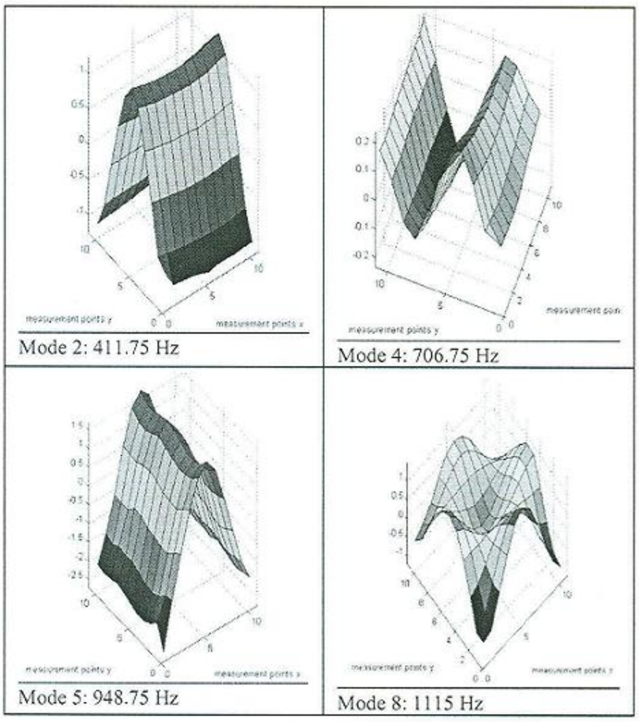

Fig. 8. Measured deflection shapes corresponding to modes 2, 4, 5 and 8 from figure 5.

To support the identification of modes an MNET [10] procedure (mixed numerical experimental technique) is used. The comparison of simulated modes and measured deflection shapes is done using the Modal Assurance Criterion [9], expressed in equation (4). This mathematical criterion compares two real vectors X and Y of the same size, producing a value between 0 and 1. If the MAC value equals 1, the vectors are identical. If zero, the vectors have nothing in common.

$$MAC_{XY} = \frac{(X^t\,Y)^2}{(Y^t\,Y)(X^t\,X)} \tag{4}$$

In equation (4), the superscript t denotes the transpose of the vector. Table 4 gives the resulting MAC values of the comparison of measured deflection shapes with the reference set of simulated mode shapes. Only the first 9 modes are considered.

TABLE 4

RESULT OF MODE SHAPE COMPARING ALGORITHM.

Simulated mode frequency (Hz)	Measured frequency (Hz)	MAC value	Deviation (%)
1	364.25	0.7469	-1.36
2	411.75	0.9818	5.4
3	619.25	0.6925	-1.98
4	706.75	0.7895	3.28
5	948.75	0.9698	-23.96
6	996.5	0.4931	-21.16
7	1079.25	0.4389	-21.32
8	1115	0.7506	-16.95
9	989.5	0.6743	3.79

Table 4 gives a good agreement for most modes. The well isolated peak with 411.75 Hz indeed has a deflection shape corresponding to mode 2, the first bending mode of the rectangular panel. This is confirmed by the good corresponding MAC value.

The measured mode at 948.75 Hz also correlates well. The other MAC values are lower, in particular those for modes 6 and 7. In general, a MAC value below 0.1 indicates that there is no similarity between the two considered vectors (or mode shapes); a MAC value below 0.4 indicates a low correlation of two modes. Obviously simulated mode 7 has no well corresponding measured deflection shape.

The other MAC values vary from 0.5 to 0.8. These values indicate a rather good correlation, although some of the measured deflection shapes are indeed coupled due to high damping, as mentioned earlier.

C. *Discussion*

A coupled mode of e.g. 2 mode shapes can mathematically be regarded as a linear combination of those 2 mode shapes. Therefore it is investigated whether a measured deflection shape is a linear combination of a few reference mode shapes. To illustrate this, the measured deflection shape with frequency 364.25 Hz is considered. Table 4 gives a MAC value of 0.7469 when the correspondence with simulated mode 1 is regarded. When the correspondence between the measured deflection shape and the first 5 reference modes is checked, table 5 is obtained.

TABLE 5

CORRESPONDENCE BETWEEN MEASURED DEFLECTION SHAPE AT 364.25 HZ AND FIRST 5 REFERENCE MODE SHAPES.

Mode	1	2	3	4	5
MAC	0.7469	0.0528	0.0023	0	0

Table 5 clearly shows that the measured deflection could be a combination of the first 3 simulated modes. There is obviously no relation between experimental mode 1 and simulated modes 3, 4 and 5.

Apart from the high damping, there is another reason why the MAC values from table 4 do not all exceed 0.9. As mentioned in section V.A the test panel is excited with random noise sound from a loudspeaker. The real sound pressure fields from the speaker are not yet taken into account.

V. MODEL UPDATING

Generally the aim of model updating is to minimise the differences between simulated and experimental results, which are in this case natural frequencies and mode shapes of a rectangular honeycomb sandwich panel with free-free boundary conditions.

A. Sensitivity analysis

The first step in performing a model updating procedure is studying the influence of every design parameter, discussed in section III, on the natural frequencies and mode shapes of the freely suspended panel. Such a study is called a sensitivity analysis. Suppose there are n_p design parameters p_j that govern a system response r_i. The change of that response due to a change of the design parameters can be expressed by equation (5).

$$\Delta r_i = \frac{\partial r_i}{\partial p_1}\Delta p_1 + \frac{\partial r_i}{\partial p_2}\Delta p_2 + \ldots + \frac{\partial r_i}{\partial p_{n_p}}\Delta p_{n_p} \qquad (5)$$

When all responses n_r are considered, equation (5) can be written in matrix form, giving equation (6).

$$[\Delta r] = [S]\,[\Delta p] \qquad (6)$$

The matrix S is referred to as the sensitivity matrix, its elements are called the sensitivity coefficients. To express the relative importance of each design parameter, the sensitivity coefficients are generally transformed to relative sensitivity coefficients and to avoid an ill conditioned sensitivity matrix all coefficients are normalised, according to equation (7).

$$s_{ij} = \frac{\partial r_i}{\partial p_j}\frac{p_j}{r_i} \qquad (7)$$

The sensitivity analysis as outlined above is carried out for the test panel with the design parameters given in table 2. For computational convenience the model with homogenised core is used here for the analysis. The analysis is performed, starting from the initial design parameter values, given in table 3. Each design parameter of the FE model is given a certain variation and the resulting change of the responses, in this case the first 10 natural frequencies, are determined. Every parameter is first varied in a wide range in order to investigate to which extent the system responses change linearly to a change of a certain parameter. For every parameter 6 variations are considered in the interval [-50%; 50%], spread symmetrically around the initial estimated parameter value. This step is necessary because the explained principle of a sensitivity analysis involves linearisation of the studied system behaviour. Only first order sensitivity is used in this study. Small parameter perturbation steps should be used in case the system behaviour is not linear.
Figure 7 shows the sensitivity matrix for the test panel, modelled as a 7 layer laminate with initial design parameters given in table 2.

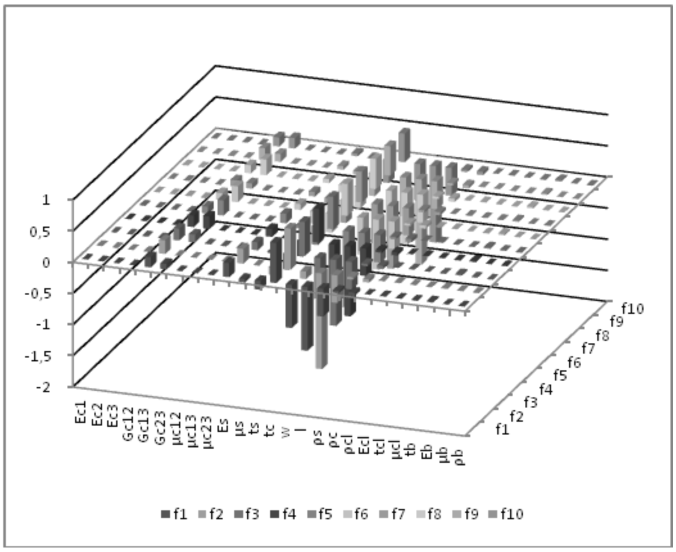

Fig. 7. Sensitivity matrix for the test panel with parameters as in table 2.

Figure 7 shows a strong variation among the different sensitivity coefficients. As mentioned earlier in the introduction, the parameters that determine the bending stiffness of the panel and the shear moduli of the homogenised core are the main parameters that govern the panel's dynamic response. For a honeycomb panel the bending stiffness per unit width increases with increasing skin thickness t_c As frequency increases, shear moduli G_{c13} and G_{c23} become more important in determining the panel's dynamic behaviour.
Along the frequency range studied, the influence of the mass densities of core and skin on the dynamic response does not change significantly.

B. Simulated versus experimentally determined natural frequencies

Table 4 shows a good correlation between simulated modes 2, 5 and 8 on the one hand, and their corresponding experimental modes on the other hand. The natural frequencies, simulated and experimentally determined, of these modes are given in table 6.

TABLE 6

COMPARISON OF THE FREQUENCIES OF 3 SIMULATED AND MEASURED MODE SHAPES.

Mode	$f_{simulation}$ (Hz)	$f_{measurement}$ (Hz)	Deviation (%)
2	434	411.75	5.40
5	729.9	706.75	3.28
8	926	1115	-16.95

Table 6 shows that the difference between simulated and measured natural frequencies increases with increasing frequency. Table 6 is used for further model updating as the 3 considered mode shapes cover the whole range of studied modes.

C. Design parameter uncertainty

The values of the different design parameters, given in table 3 are not exact. Some of them are given by the manufacturer's specification, others are estimated. The precise determination of a parameter value is sometimes impossible. In that case, a range should be specified for the parameter. When many samples of a population are available, a probability interval and a probability density function should be defined. In most cases a uniform or a normal distribution is used to consider the uncertainty of a specific parameter. However, the exact distribution of a parameter is seldom known. For convenience, a uniform distribution is considered here. For each studied design parameter, the probability interval is centred around its corresponding estimated value from table 3. For different kinds of parameters, relative interval widths vary as given by table 7.

TABLE 7

RELATIVE PROBABILITY INTERVAL WIDTHS.

Parameter description	Relative probability interval width (%)
Overall panel dimensions	1
Skin material properties	2
Core outer layer properties	10
Homogenized core material properties	20
Bonding layer material properties	50

The overall panel dimensions are easily be measured. The probability interval width of these parameters is therefore set equal to the measurement accuracy of 1%.

With this test panel the skin faces are made from steel. Its mass density and stiffness were experimentally determined and compared to manufacturer's specifications, leading to the proposed interval width of 2%. Mass density and stiffness were also experimentally determined for the core material. In this case, comparison with values from the panel manufacturer gave a tolerance of 10%.

The elastic properties of the homogenized honeycomb core were calculated from FE models. These were compared to analytically determined values, calculated with Gibson and Ashby's [2] formulas. This approach lead to an interval width of 20%.

For the properties of the bonding layer material just some estimated values were available. These parameter values were not determined experimentally so no comparison could be carried out. This leads to the large interval width of 50%.

The considered interval widths of core and bonding layer parameters are rather wide. It is perhaps questionable if the linearised parameter influence, resulting from the sensitivity analysis (see section VI A), approaches the real parameter influence in an acceptable way. As an example the relative sensitivity of mode 2 to the homogenised core shear modulus G_{c13} is shown in figure 8.

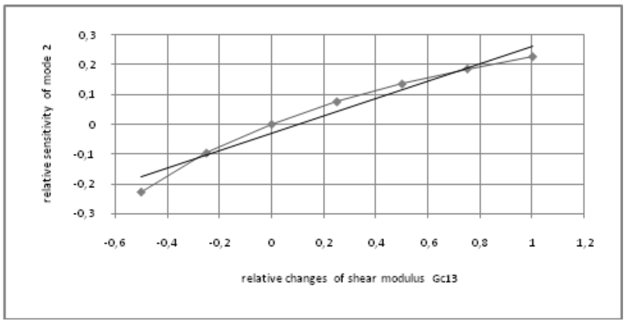

Fig. 8. Mode 2 relative sensitivities for relative changes of shear modulus G_{c13}. A Linear least squares approximation is added.

From the linear least squares approximation in figure 8 it is obvious that the influence of shear modulus G_{c13} on the frequency of mode 2 is not linear. In this case however, the horizontal axis covers a relative change of 100 % which is far more than the parameter variations considered in this study. If a symmetric interval of 20 % around the initial value of G_{c13} is considered, a linear approximation is very good in that interval (correlation 0,9997), as shown in figure 9.

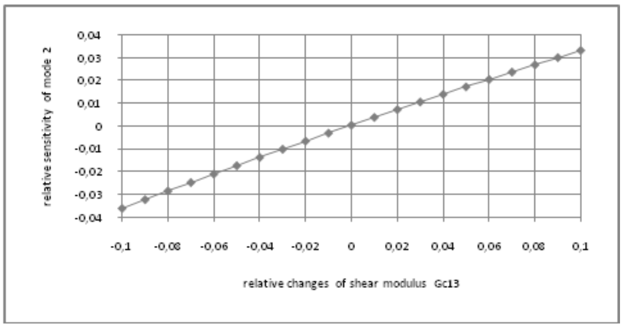

Fig. 9. Mode 2 relative sensitivities for changes of shear modulus G_{c13} in the interval [-0.1;0.1].

For the other design parameters similar conclusions can be drawn. If the probability intervals are kept small enough, the linearisation process of the sensitivity analysis will not create severe errors. In future research more precise sensitivity analysis will be carried out. For each design parameter a more correct interval width and probability function will be taken into account.

D. Model updating: results and discussion

For the test panel with design parameters given in table 2 and probability intervals in table 8 the results of the sensitivity analysis, as described in previous section C, are used to perform a model updating procedure. It is outlined in figure 10.

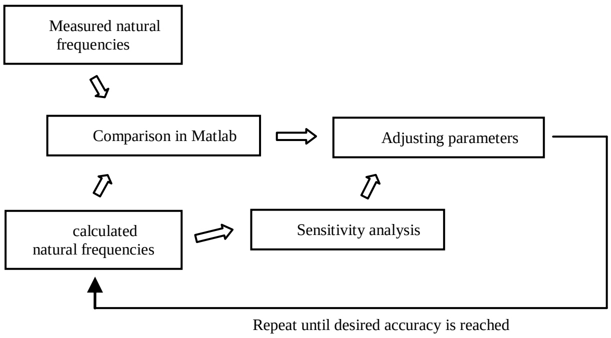

Fig. 10. Procedure for model updating.

When an infinitely large population of a certain parameter is considered, the mean value of the population is equal to the real expected parameter value. The smaller the size of the population, the more the population mean differs from the real expected value. To limit the error e between the calculated mean value and the real expected value, the population should have a minimum size n_{min}. According to Montgomery [16], equation (8) calculates the required population size in case of a Normal parameter distribution.

$$n_{min} = \left(\frac{3 \cdot \sigma_{ini}}{e}\right)^2 \qquad (8)$$

In equation (8), σ_{ini} is the standard deviation of the initial maximum difference between calculated and measured frequency. In this study, e is the allowed absolute error on the calculated natural frequencies mean values. In this case 1 % of the initial maximum error is taken.

The principle of Monte Carlo simulations is that for a set of parameters random values are generated, taking into account the probability distribution for each parameter. In this case a uniform distribution is taken for every design parameter. For every set of random parameter values the output of the system, in this case natural frequencies of a honeycomb panel, is calculated. Monte Carlo simulations are a very useful tool in determining the probability distribution on the output of a process or the response of a system, considering input parameter uncertainty. In order to get good knowledge of the output distribution, the Monte Carlo simulations should be carried out a number of times. The higher the number of iterations, the more accurate the distribution prediction is.

In this study, each set of design parameters is used to calculate the values of the natural frequencies of modes 2,5 and 8. For this, the results of the sensitivity analysis are used.

The number of iterations used here is n_{min}, calculated with equation (8). As a result of these Monte Carlo simulations, the distributions of the 3 considered natural frequencies are calculated, taking into account design parameter uncertainty.

The goal of the updating procedure in this study is to minimize the difference between calculated and measured natural frequencies by tuning the different design parameters. Here the updating procedure is combined with Monte Carlo simulations. In this way, not only a set of optimised design parameter values and their corresponding natural frequencies are obtained, but also the probability distribution of the calculated natural frequencies is estimated.

As a result of the model updating procedure, improved values for the 3 natural frequencies of modes 2, 5 and 8 are calculated. These calculated values are in fact the mean values from a distribution, which is known from the Monte Carlo simulations. The set of design parameters, yielding these frequencies, represent the optimised design parameter mean values.

In this case the whole updating procedure is carried out 3 times. For simplicity every cycle the same relative probability interval width is taken for a specific parameter.

For every kind of distribution there are certain parameters that compare the shape of the distribution to the shape of a normal distribution. In this study the skewness γ and the kurtosis δ are used. They are calculated with equation (9).

$$\gamma = \frac{\tilde{\mu}_3}{\sqrt{\tilde{\mu}_2^3}} \quad \text{and} \quad \delta = \frac{\tilde{\mu}_4}{\tilde{\mu}_2^2} \qquad (9)$$

In equation (9), $\tilde{\mu}_i$ is the i-th moment of the distribution. A positive skewness means that the distribution is asymmetric to the right. The skewness of a normal distribution is therefore equal to zero. The kurtosis of a normal distribution is 3. A kurtosis value greater than 3 means that the distribution is more pointed than a normal distribution.

For the calculated natural frequencies of the 3 studied modes, table 8 gives the mean value, standard deviation of a Normal distribution, skewness and kurtosis after the third iteration.

TABLE 8

DISTRIBUTION PARAMETERS FOR THE 3 CALCULATED NATURAL FREQUENCIES AFTER THE THIRD ITERATION

Mode	Mean frequency (Hz)	Standard deviation (Hz)	Skewness	Kurtosis
2	410.97	13.69	-0.0096	2.93
5	706.26	23.46	-0.0092	2.92
8	1011.08	33.73	-0.010	2.94

The skewness and kurtosis values indicate that the calculated frequency distributions are nearly Normal.

An overview of the obtained relative errors on the 3 natural frequencies is given in table 9 for each iteration.

TABLE 9

RELATIVE ERRORS ON THE 3 NATURAL FREQUENCIES DURING THE MODEL UPDATING PROCEDURE FOR THE FIRST 3 ITERATIONS

Mode	$Error_{initial}$ (%)	$Error_{iteration1}$ (%)	$Error_{iteration2}$ (%)	$Error_{iteration3}$ (%)
2	5.40	3.23	1.16	-0.19
5	3.28	2.11	0.87	-0.07
8	-16.95	-13.73	-10.41	-9.32

As is indicated in table 9, the first iteration yields the most significant error reduction. Iterations 2 and 3 still produce some error reduction. It is clear however that in spite of a further increase in the number of iterations the error with mode 8 will stay around 10%. This indicates that, with the current set of design parameters and FE model, the errors cannot be substantially further reduced. There are a number of reasons for this.

In the updating procedure, the frequencies of the experimental modes are assumed to be correct. If there are any errors on the identified natural frequencies, the updated model may not converge to the measured model.

A second reason could be that some initial values of some design parameters were fully estimated, e.g. the stiffness and thickness of the bonding layer. Earlier it has been mentioned that, with increased frequency, the importance of shear deformation increases. As the bonding layer is mainly shear loaded, an underestimation of e.g. the glue stiffness leads to an underestimation of calculated natural frequencies. According to the results of the previously described sensitivity analysis an underestimation of the glue stiffness of 10% leads to an underestimation of 0.23 % of the natural frequency from mode 2. Hence it seems unlikely that a wrong estimation of the glue parameters could explain the large errors in the calculated frequencies for modes 2 and 4. Future research will be done in this area, using more and larger test panels. Also measurements will be carried out to determine honeycomb core elastic properties more accurately. Further work must also be done on the specific variability of every design parameter.

VI. CONCLUSIONS

In this article the composition of typical honeycomb sandwich panels is discussed, in particular panels with a thermoplastic ThermHex core. The different design parameters of such panels are outlined.

Two ways of building up finite element models in commercial software are outlined. Advantages and shortcomings of both methods are discussed.

The experimental determination of mode shapes and their natural frequency has been discussed thoroughly, emphasising on the fact that high structural damping may lead to complications in this area.

The different steps, leading to the updating of FE models, are discussed extensively. These include design parameter uncertainty, sensitivity analysis and the use of Monte Carlo routines. The results of the application on a test panel are discussed.

The real uncertainties on the different design parameters will be thoroughly studied, using an elaborate amount of test honeycomb panels. Special attention will be made to the true elastic properties of the bonding layer and to the frequency dependency of elastic material properties.

REFERENCES

[1] Dan Zenkert, *An introduction to Sandwich Construction*, Emas Publishing, 1996.

[2] L.J. Gibson & M.F. Ashby, *Cellular solids,* Pergamon Press, 1988.

[3] E. Nilsson & A.C. Nilsson, *Prediction and measurement of some dynamic properties of sandwich structures with honeycomb and foam cores*, Journal of sound and vibration, (2002) 251(3), 409-430.

[4] R.D. Blevins, *Formulas for natural frequency and mode shape*, Krieger Publishing Company 1984.

[5] P. Topdar, *Finite element analysis of composite and sandwich plates using a continuous inter-laminar shear stress model,* Journal of sandwich structures and materials, 2003, 5, 207.

[6] Qunli Liu, *Role of anisotropic core in vibration properties of honeycomb sandwich panels,* Journal of thermoplastic composite materials, 2002, 15;23.

[7] Qunli Liu, *Effect of soft honeycomb core on flexural vibration of sandwich panel using low order and high order shear deformation models,* Journal of sandwich structures and materials, 2007; 9; 95.

[8] Qunli Liu, *Prediction of natural frequencies of a sandwich panel using thick plate theory,* Journal of sandwich structures and materials, 2001;3;289.

[9] W. Heylen, S. Lammens, P. Sas, *Modal analysis Theory and testing*, KU Leuven, 2003.

[10] Tom Louwagie, *Vibration – based methods for the identification of the elastic properties of layered materials,* Doctoraatsproefschrift D/2005/7515/80, 2005.

[11] S. Adhikari en J.Woodhouse, *Identification of damping: part I, viscous damping,* Journal of sound and vibration (2001) 243(1) 43-61.

[12] Junhong Park, *Transfer function methods to measure dynamic mechanical properties of complex structures,* Journal of sound and vibration 288 (2005) 57-79.

[13] Fang-Lin Huang, *A new approach to identification of structural damping ratios*, Journal of sound and vibration 303 (2007-) 144-153.

[14] L. Valdevit, *Structurally optimized sandwich panels with prismatic cores,* International journal of solids and structures 41 (2004) 5105-5124.

[15] Meifen He, *A study on composite honeycomb sandwich panel structure*, Materials and design 29 (2008) 709-713.

[16] Montgomery Douglas, Design and analysis of experiments, Arizona State University, 1996